Sensitivity analysis of the shrinkage-swell curve

This document constitutes a complementary report on the sensitivity analysis of the shrinkage-swell curve equation adopted in the paper titled “A simple method to determine soil-water retention curves of compacted active clays” at the journal Transportation Geotechnics. It is suggested to refer to the main publication regarding the nomenclature adopted in this document and details on the experimental data adopted.

The following equation is being suggested as a possible way of describing the shape of shrinkage-swell curves (SSC) represented in terms of water ratio (e_w) versus void ratio (e). This equation has two fitting parameters referred as a and b. The minimum void ratio reached upon drying, i.e., the void ratio in the residual state is also required and is represented by e_{min}.

$$e = e_{min} + (e_w + a)^b$$

Varying the parameter a changes the slope of the SSC in the residual shrinkage zone, while parameter b changes the position of the apparent shrinkage limit as observed in the figure below (Figure 1).

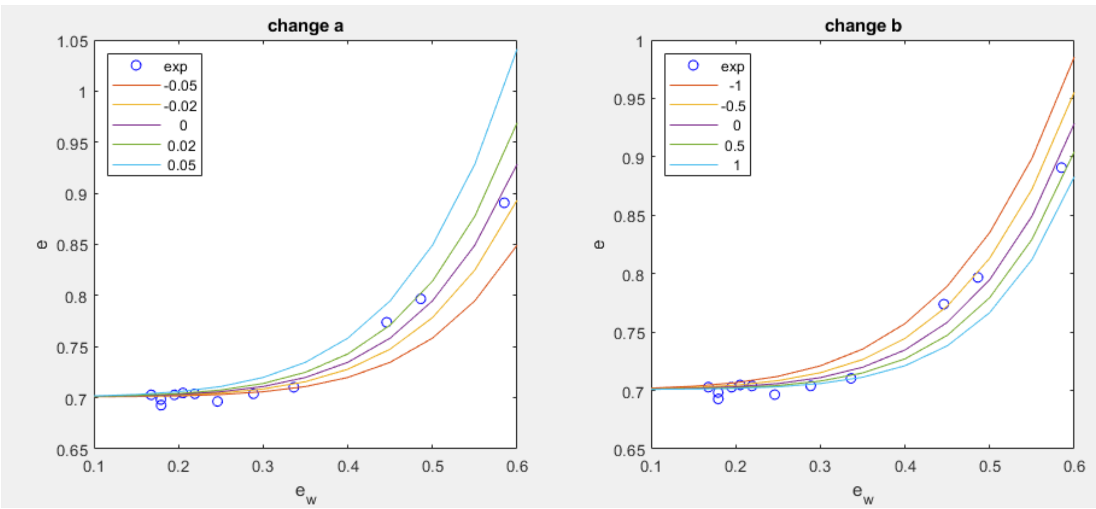

Figure 1 – Effect of varying the parameter a and b on the shape of the SSC.

The equation above mentioned was fitted to each drying and wetting phase over the six cycles for five samples of two different plasticity clays. Hence, a total of 120 curves were fitted. The fitting was obtained using MatLab functions using the least-square method. An example is presented below of obtained fitting for the SSCs of the first three phases of the drying-wetting cycles of all the samples of both clays (Figure 2).

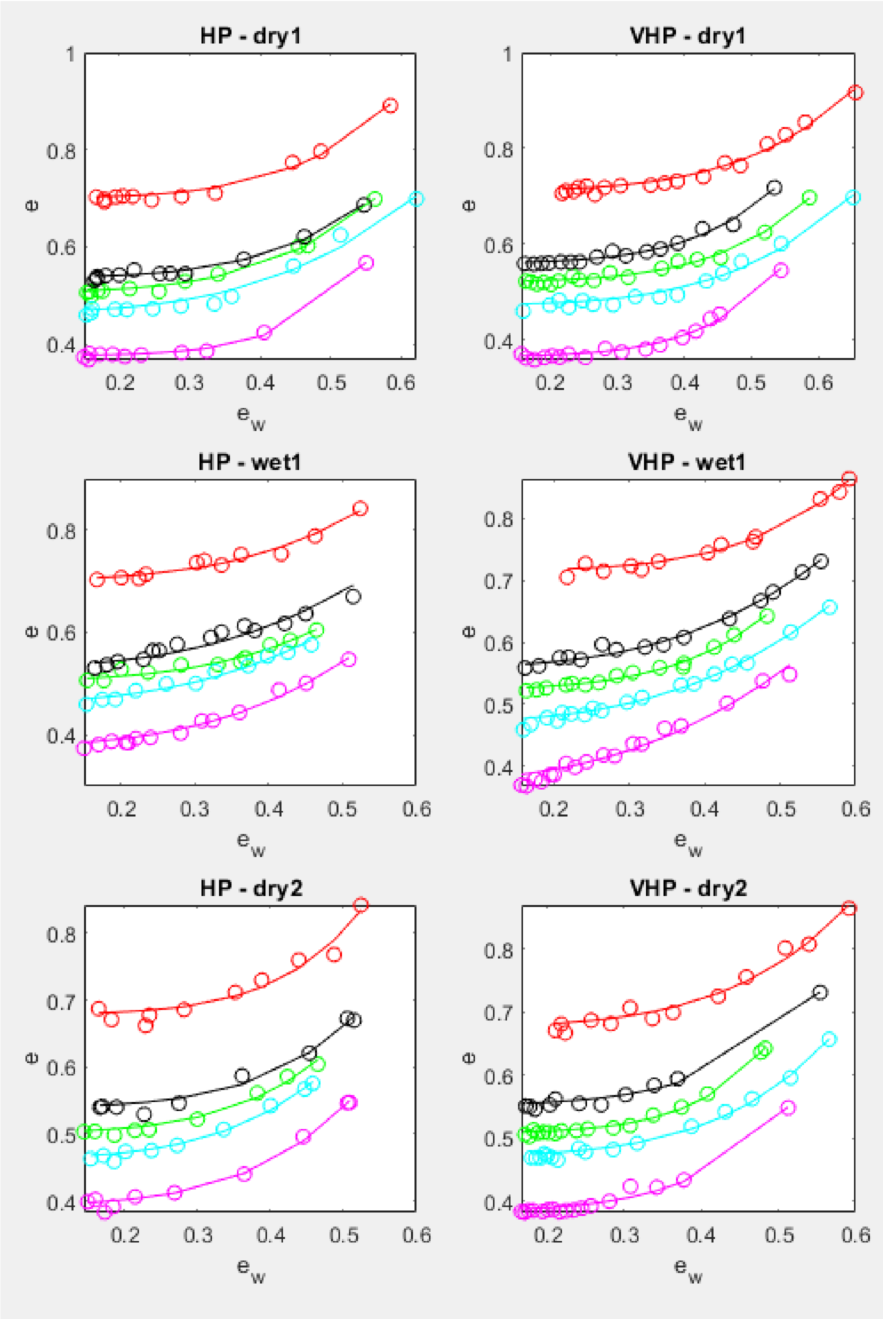

Figure 2 – Obtained fitting of the SSC of phases drying 1, wetting 1, and drying 2 (legend: red – loose; green – optimum; pink – dense; blue – wet; black - dry).

The obtained fitting presented reasonable values of coefficient of determination as observed in Figure 3. The coefficient of determination was always above 0.92 and often above 0.96, which constitutes an excellent fitting.

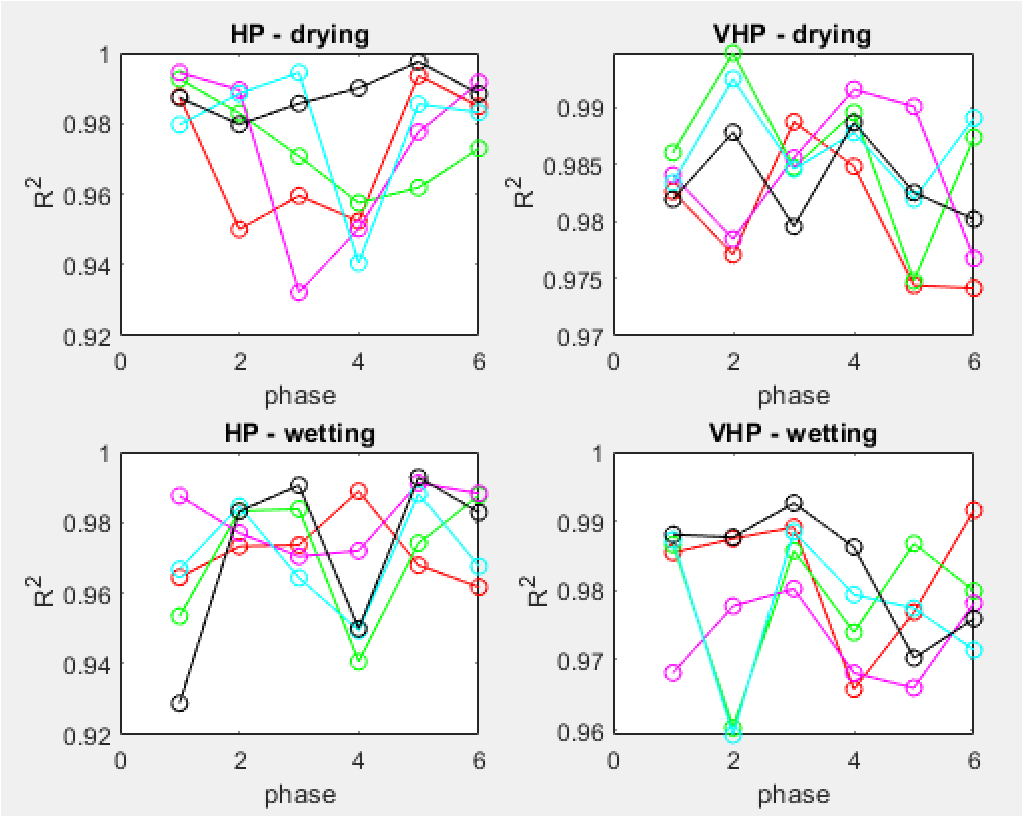

Figure 3 – Coefficient of determination of each of the phases and of both clays (legend: red – loose; green – optimum; pink – dense; blue – wet; black - dry).

The study focuses on the effect of compaction conditions on the evolution of the hydraulic properties with cycles of wetting and drying. Hence, there is an interest in finding a pattern in the evolution of the fitting parameters of the SSC. The figure below shows these parameters with cycles of wetting and drying; however, it was observed that no trend in the evolution of these parameters is evident (Figure 4). Moreover, the range of variation of these parameters is quite substantial, for which this sensitivity analysis was found necessary.

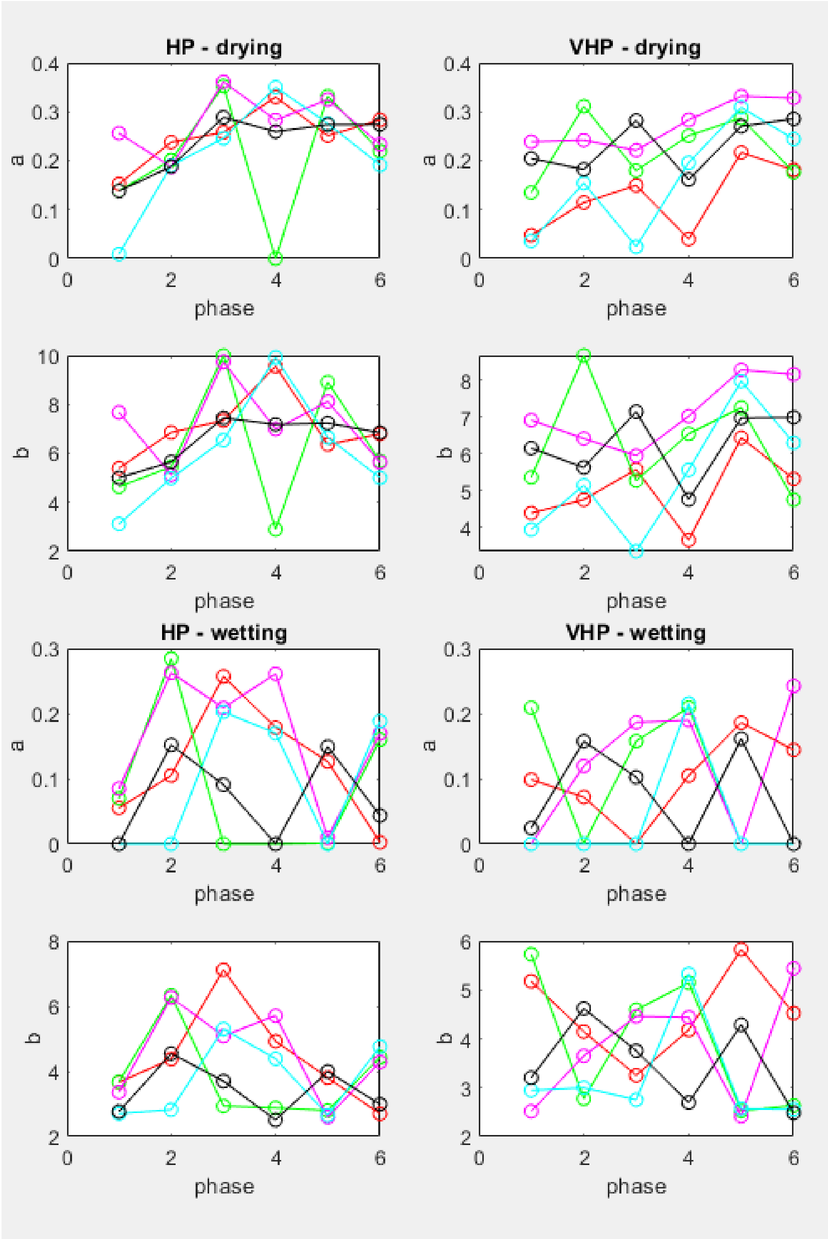

Figure 4 – Evolution of the fitting parameters of the SSC with drying-wetting cycles of samples compacted at different conditions (legend: red – loose; green – optimum; pink – dense; blue – wet; black - dry).

The sensitivity analysis was performed by perturbing the fitting parameters by one standard deviation. The standard deviation refers to the standard deviation of each fitting parameter for the entire population of fitted parameters reported in Figure 4. This standard deviation was added and subtracted from the fitted parameter to obtain two perturbed parameters. These perturbed parameters were used to produce an estimated void ratio which is then compared to the actual void ratio as presented in the Figure 5

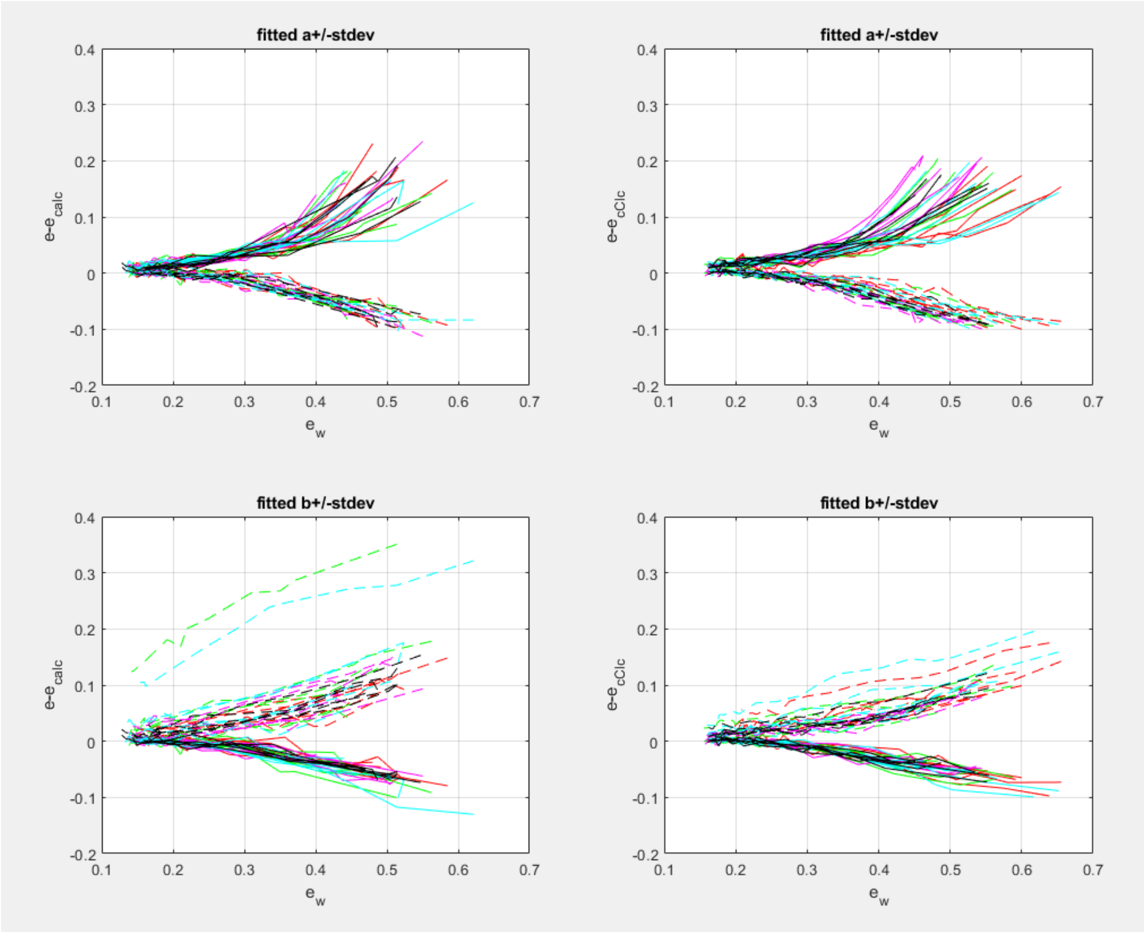

Figure 5 – Deviation of the estimated void from the actual void ratio in which the estimated void ratio is obtained adopting perturbed fitting parameters of one standard deviation.

It was observed that the deviation of e_{calc} from the real void ratio (e) varied between -0.15 and 0.25, approximately, for disturbances to the value of a and b. It was also observed that the value of $e - e_{calc}$ when the value of b was disturbed was greater than when the value of b was disturbed over a wider range of water ratio (Figure 5). However, when the value of a was disturbed, an increase in the $e - e_{calc}$ value is observed for high values of e_w with a tendency that is almost exponential, for which it was opted for allowing the parameter a to vary, which should minimize errors in the wetter state of the soil (i.e., in the low suction) when adopting such equation in the context of the mentioned publication.

It is worth pointing out that fixing one of the parameters is not actually required in the context of the publication. However, having a single parameter to fit instead of two with increasing number of cycles can be an advantage for further research and of the interest of the reader.

A new fitting of SSC equation was then performed in which the value of the parameter b was fixed constant as the average value obtained in the previous fitting for each of the clays. The fitted values are presented below and it is now observed that the parameter a varying with increasing number of cycles (Figure 6). Nonetheless, the coefficient of determination did not decrease significantly and a good fitting of the SSC to the experimental is still obtained as the value of the coefficient of determination is almost always above 0.90 (Figure 7). A visual example of the quality can also be assessed through the SSCs presented below (Figure 8).

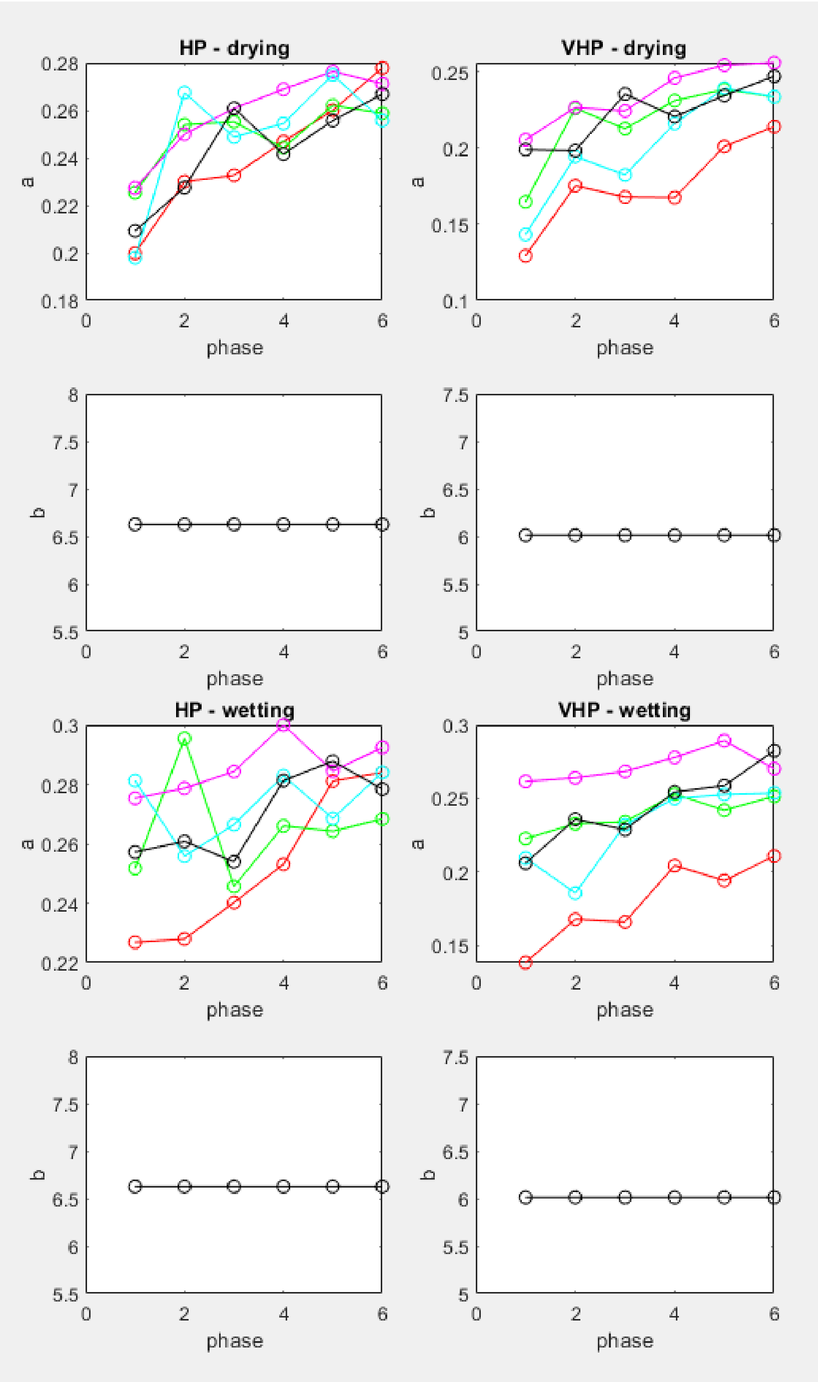

Figure 6 – Evolution of the fitting parameters a and b assuming that b remains constant (legend: red – loose; green – optimum; pink – dense; blue – wet; black - dry).

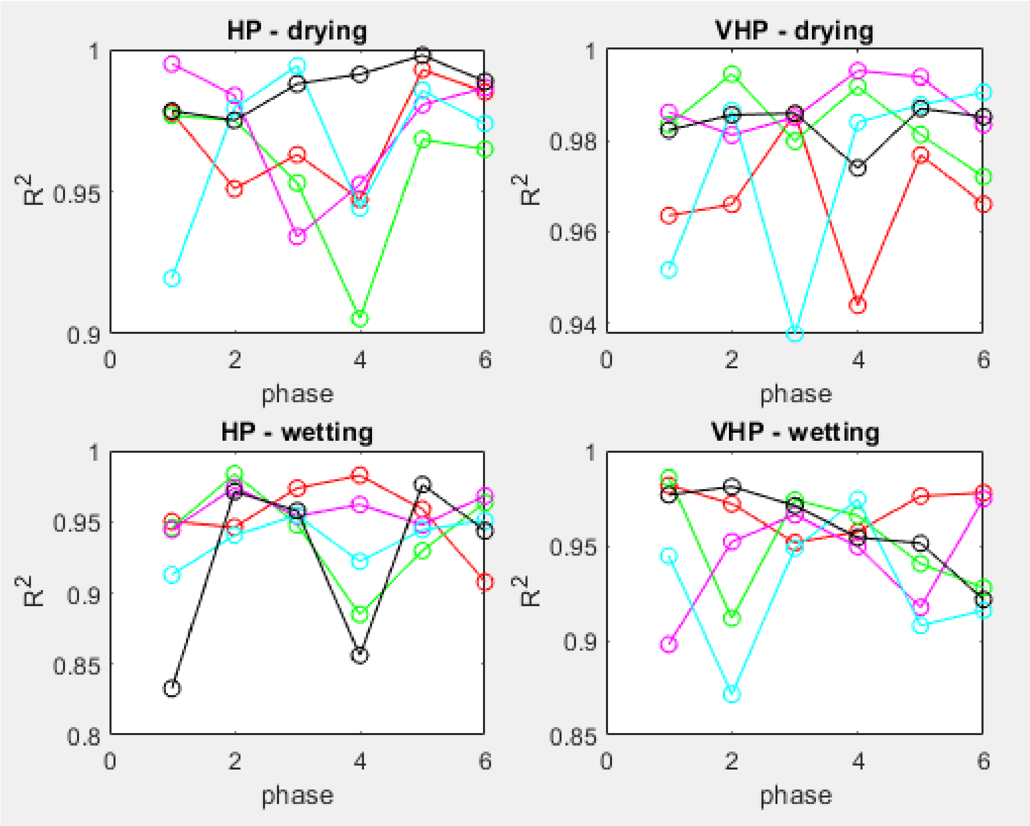

Figure 7 – Coefficient of determination of each of the phases and of both clays assuming a constant value for parameter b (legend: red – loose; green – optimum; pink – dense; blue – wet; black - dry).

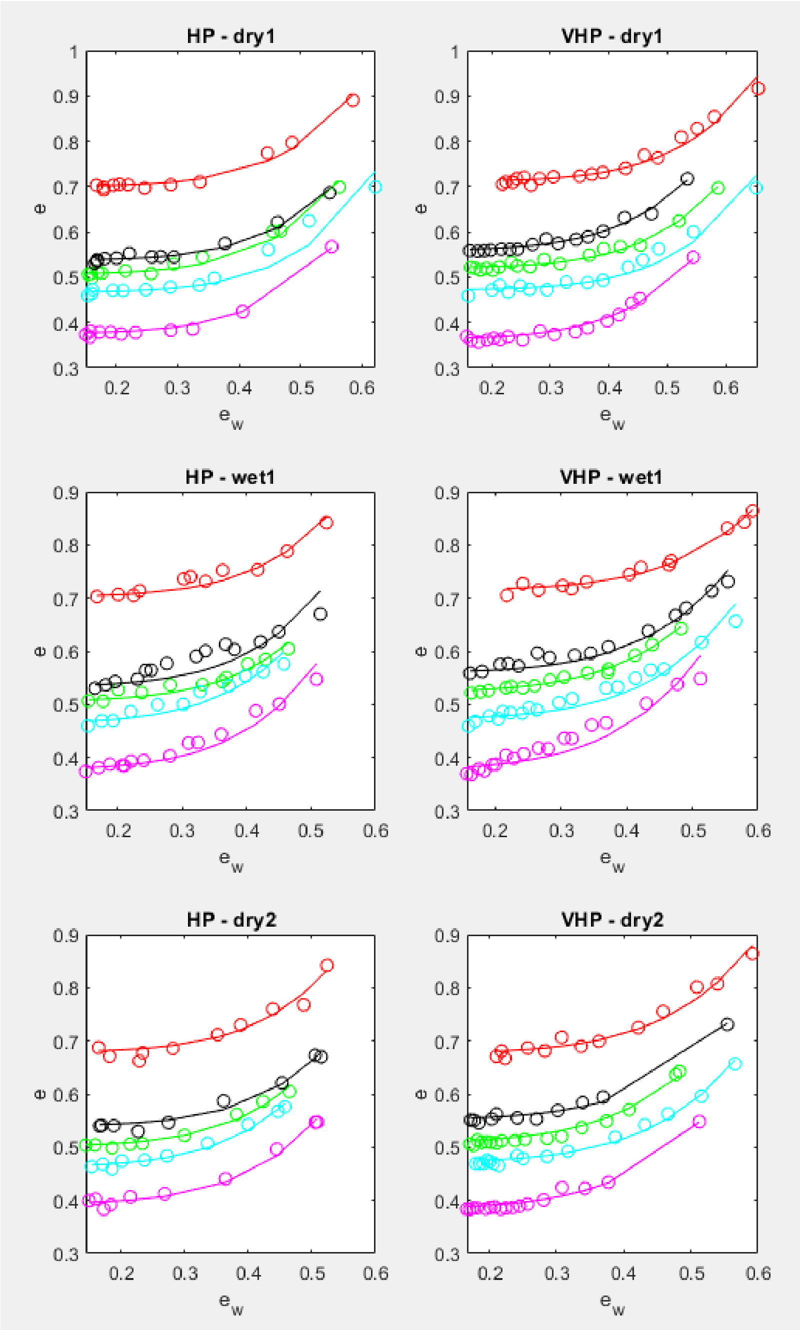

Figure 8 – Obtained fitting of the SSC of phases drying 1, wetting 1, and drying 2 assuming a constant parameter b (legend: red – loose; green – optimum; pink – dense; blue – wet; black - dry).

Acknowledgements:

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